Differentiation of Instructional Methods

Exponential Station Work

If you look at Exponential Functions Unit Expectations Cover Sheet, you will see the skills that are addressed in the station work.

At the table are station cards for you to try out the problems.

IXL

IXL is an on-line skills practice program. Today, I brought 7 IXL logons for people to try it out. You can play with the sections that I have my class use in this unit (See Exponential Functions Unit Expectations Cover Sheet), or you can try out different sections that you might be interested in for your class.

See the index card at the computers for the website, username and password

If the tabs come up as Grade specific, then Level J is the same as 8th grade.

J-F8....Level J (or tab J)....Problem F.8

Desmos

I have created a class for a specific Desmos activity that you might choose to use in your class. It could be used at many different levels of algebra and as a grabber for many different units. I used it just to introduce functions as a whole. You can use Desmos on any of the computers (or on your own). Not sure how the IPAD will handle it.

See the index card at the computers for the website and class code.

Desmos accounts are FREE...

Re-Instruction on Exponent Laws

I have created instructional videos on the hows and whys of exponent laws. You can view them at:

http://www.theparkerschool.org/pages/Francis_W___Parker_Charter_Esse/Classes/D2MST60

(or computers might default to that page) and then Select the Fall Forum Extreme Differentiation Folder on the Side Bar

Open the 1 Differentiated Instruction Folder and select one of the following videos:

<table>
<thead>
<tr>
<th>7a Exponent Laws Multiplication Division.mp4</th>
<th>7b Exponent Laws Power Law.mp4</th>
<th>7c Exponent Laws Distributive Law.mp4</th>
</tr>
</thead>
<tbody>
<tr>
<td>7d Exponent Laws Zero Power Law.mp4</td>
<td>7e Exponent Laws Negative Exponent Law.mp4</td>
<td>7f Exponent Laws Mix It Up.mp4</td>
</tr>
</tbody>
</table>
**Ex**treme Differentiation
In the Math Classroom

Dawn Crane
Math Specialist Teacher
Francis W. Parker Charter Essential School
Dorchester, MA

**Goal of this Workshop**

To learn/experience some methods of differentiated instruction in mathematics
To learn/experience some strategies for tracking student progress
To learn/experience some methods of differentiated assessment

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Who Am I?

Dawn Crane
Francis W. Parker Charter Essential School

- Currently teaching Division 2 (9th/10th) grade Math
- Have taught Division 1 (7th/8th) and Division 3 (11th/12th)

- School Philosophy (10 common principles)
  - Less is More
  - Depth Over Breadth
  - Student as Worker/Teacher as Coach
  - Demonstration of Mastery

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What Do I Call It

**EX**TREME Differentiation?

- Current Program Structure
  - Students spend ~2 years in a "Division"
  - Class contains 1st and 2nd year students
  - Variety of skill levels even within the 1st and 2nd year
  - When needed, students who are in 11th grade or 3rd year continue in Division.
  - Occasionally an 8th grader will advance to Division 2 early
  - Overall, the class could contain 8th-11th graders, each at different levels of previous knowledge and pace of learning.
  - Classes are heterogeneously grouped...no tracking
  - The majority of students on an IEP are included in the general education classroom.
• Hook...whole class activity to peak students' curiosity and interest

• Initial Learning...whole class activities done in table groups

• Students are provided a Unit Expectations Overview that tells them the skills they will learn and the options they have for learning and sometimes demonstrating that proficiency

• Understanding Check...students hand in class work...teacher reviews and groups students accordingly

• Practice...once basic concepts are introduced, students practice...self-paced (with teacher guidance)

• On-going formative assessment while practicing informs need for re-instruction and going further work.

• Summative Assessment....1 or 2 per unit, focusing on 1 or more "skill" areas

• How far a student completes and assignment (adjusting quantity)

• Having extensions available that encourage more critical thinking

• Grouping and regrouping based on need

• Allowing/encouraging group conversation about the work

• Using exit slips and formative assessment to group and regroup and decide when to provide re-instruction and to whom

• Having simplified and more complex versions of problems

• Providing some choice

• Minimizing homework...moving work into the classroom so students can receive more guidance and prompting.

• Providing alternate ways to demonstrate understanding of concepts. Quizzes, IXL, oral revision, on the spot working of problems

• Providing "complexity" options within the assignment

• Providing problems with multiple entry and exit points

• Giving options for how the work is presented

• Providing options based on interest.
How many dominoes would it take to knock over an Empire State Building sized domino?

(A smaller domino can topple a domino that is up to 1.5 times larger in every dimension.)

Watch the video on our Class Web Page to see the scenario....

**(I download the video to my class webpage... I have put the Dan Meyer Link here for you)**

http://threeacts.mrmeyer.com/domino-skyscraper/

Given the information provided in the video and the back of this page, begin to solve the problem...

Taking it Further...

Suppose that the second domino is placed 2.5 mm away from the first (smallest) domino, and the distance between each domino also grows by a factor of 1.5.

How much space would you need in order to knock down the Empire State Building size domino? A city block? An airport runway? The entire United States?
Pay It Forward

https://www.youtube.com/watch?v=DGNobEDq5JU

In the popular book and movie, Pay It Forward, 12-year-old Trevor McKinney gets a challenging assignment from his social studies teacher.

Think of an idea for world change, and put it into practice! Trevor came up with an idea that fascinated his mother, his teacher, and his classmates.

He suggested that he would do something really good for three people. Then when they would ask how they can pay him back for the good deeds, he would tell them to “pay it forward” – each doing something good for three other people.

Trevor figured that those three people would do something good for a total of nine others. Those nine would do something good for 27 others, and so on. He was sure that before long there would be good things happening to billions of people all around the world.
PAY IT FORWARD Analysis Questions

4. How does the number of good deeds at each stage grow as the tree progresses? How is that pattern change shown in the plot of the data?

5. How many stages of the Pay It Forward process will be needed before a total of at least 25,000 good deeds will be done? (How do you know?)

6. Write a rule (equation) relating Number of Good Deeds (N) to the Stage of Process (x). This rule could be used to model the Pay It Forward Process in which each person does good deeds for three other new people. Show work below:

7. Write an "N=..." rule (equation) that would show the number of good deeds at stage number x if each person in the process does good deeds for two others.

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Is it possible for one idea to change the world?

1. Which of the graphs below do you think is most likely to represent the pattern by which the number of people receiving Pay It Forward good deeds increase as the process continues over time?

2. What is your best guess about the number of people who would receive Pay It Forward good deeds at the tenth stage?

3. How many people would receive a Pay It Forward good deed at each of the next several stages?
   a. Complete the table below that shows the number of people who will receive good deeds at each of the next seven stages of the Pay It Forward process. Then plot the data on a graph. Make sure you have accurate axes labels and scales.

<table>
<thead>
<tr>
<th>Stage of Process</th>
<th>Number of Good Deeds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
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<td>3</td>
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<td>10</td>
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</tbody>
</table>

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OVER →
Exponential Applications

% Growth

Population Growth (% growth)

Brazil is the most populous country in South America. In 1995, its population was about 161 million. It is growing at a rate of 1.2% per year.

- Create a table, graph, and equations (recursive and explicit) to model this situation.
- What would the population be in 2013? What was the population in 1990?
  (Show how to find your answers using the table and the explicit equation.)

- Explain how you can tell the situation is exponential:
  From the table.
  From the graph.
  From the scenario.
  From the explicit equation.

<table>
<thead>
<tr>
<th>Years After 1995</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Y</td>
</tr>
</tbody>
</table>
Exponential Applications

% Decrease

**Key Learning**

Medicine Decay (\% decrease)

Diabetes, a disorder in which the body cannot metabolize glucose properly, affects people of all ages. In 1995, there were about 8 million diagnosed cases of diabetes in the United States. It was estimated that another 8 million cases remain undiagnosed.

A typical pattern of insulin decrease is 5\% per minute.

- Create a table, graph, and equations (recursive and explicit) to model this situation.
- If a person begins with 10 units of insulin upon injection, how many units of insulin will be in the bloodstream after 15 minutes? 30 minutes?
  - Show how to find your answers using the table and the explicit equation.
- At what time will half of the insulin be left?
- Explain how you can tell the situation is exponential:
  - From the table.
  - From the graph.
  - From the scenario.
  - From the explicit equation.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Insulin Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td></td>
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</tr>
</tbody>
</table>
On the Brazilian Population Problem I am...

A. Finished (created table and equation)

B. In Process (created/began table, haven’t yet got equation)

C. Not Certain (I have a table but my numbers don’t make sense)

D. Don’t know where/how to start

E. I am still working on ____________________________ problem

F. Other....tell me.....
Exponential Functions Unit Overview
Infection, Disease & Immunity
Spring 2015

By the end of this unit, a proficient student will be able to:

<table>
<thead>
<tr>
<th>Exponential Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1 Evaluate integer exponents</td>
</tr>
<tr>
<td>E2 Evaluate zero &amp; negative exponents</td>
</tr>
<tr>
<td>E3 Apply multiplication/division exponent laws</td>
</tr>
<tr>
<td>E4 Apply power/distribution exponent laws</td>
</tr>
<tr>
<td>E5 Recognize an exponential function from a graph</td>
</tr>
<tr>
<td>E6 Recognize an exponential functions from a table</td>
</tr>
<tr>
<td>E7 Recognize an exponential functions from an equation</td>
</tr>
<tr>
<td>E8 Generate exponential equations from tables, graphs, scenarios</td>
</tr>
<tr>
<td>E9 Solve exponential equations using tables &amp; graphs</td>
</tr>
</tbody>
</table>

In addition, students will understand:
- The role of a negative exponent in modeling scenarios
- The role of multiplication and division in modeling scenarios

Some students will be able to:
- Identify/explore how changes in parameters are reflected on the coordinate plane
- Identify/explore how exponent laws can explain some of these shifts.

Station Work (Covers generating, using, and solving exponential functions) [E5-E9]

<table>
<thead>
<tr>
<th>Station 1</th>
<th>Generate Equations from Scenarios (Whole #'s &amp; Fractions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station 2</td>
<td>Generate Equations from Scenarios (% Increase)</td>
</tr>
<tr>
<td>Station 3</td>
<td>Generate Equations from Scenarios (% Decrease)</td>
</tr>
<tr>
<td>Station 4</td>
<td>Generate Equations from Scenarios (Mixed)</td>
</tr>
<tr>
<td>Station 5</td>
<td>Solving Exponential Equations Graphing</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Station 6</th>
<th>Is It Exponential? Why or Why Not?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station 7</td>
<td>Create a Data Table from a Scenario</td>
</tr>
<tr>
<td>Station 8</td>
<td>Create an Equation from a Data Table</td>
</tr>
<tr>
<td>Station 9</td>
<td>Create an Equation from a Graph</td>
</tr>
<tr>
<td>Station 10</td>
<td>Growth or Decay? Factor and Rate?</td>
</tr>
<tr>
<td>Station 11</td>
<td>Using Equations</td>
</tr>
</tbody>
</table>

Between stations 1-4 you need to show at least 3-% increase and at least 3-% decrease problems. You are responsible for completing stations 5-11.

There are also extension stations to challenge you further once you complete the required learning.

IXL & Additional Problem Sets (Covers Exponent Laws and Critical Thinking around Exponent Laws) [E1-E4]

<table>
<thead>
<tr>
<th>IXL</th>
<th>Exponent Laws</th>
</tr>
</thead>
<tbody>
<tr>
<td>J-F8</td>
<td>Multiplication with Exponents</td>
</tr>
<tr>
<td>J-F9</td>
<td>Division with Exponents</td>
</tr>
<tr>
<td>J-F10</td>
<td>Multiplication &amp; Division with Exponents</td>
</tr>
<tr>
<td>J-F11</td>
<td>Power Rule</td>
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<tr>
<td>J-F5</td>
<td>Exponents with Decimal &amp; Fractional Bases</td>
</tr>
<tr>
<td>J-F6</td>
<td>Understanding Negative Exponents</td>
</tr>
</tbody>
</table>

You can begin with F10. If you are having difficulty there, then go back to F8 & F9.

Your Smart Score should be between 70 and 100 to be ready for the quiz. Unless you are really having difficulty with the material, you should be aiming for >85.
**Exponential Functions Stations**

Station 1: Generate Equations from Scenarios (Whole # & Fractions) *(Need at least 3 growth and 3 decay between stations 1-4)*

<table>
<thead>
<tr>
<th>Prob #</th>
<th>Prob #</th>
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<tbody>
<tr>
<td></td>
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</table>

Station 2: Generate Equations from Scenarios (% Increase) *(Need at least 3 growth and 3 decay between stations 1-4)*

<table>
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<tr>
<th>Prob #</th>
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</table>

Station 3: Generate Equations from Scenarios (% Decrease) *(Need at least 3 growth and 3 decay between stations 1-4)*

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<tbody>
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Station 10: Growth or Decay? Factor and Rate? (3)
# Exponential Stations Checklist

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</tr>
</tbody>
</table>

*This is how I keep track of kids progress through station work*
Multiplication and Exponents

*Developing Exponent Laws*

1. Rewrite each product below in expanded form and then rewrite it in exponential form with a single base.

<table>
<thead>
<tr>
<th>Expanded Form</th>
<th>Exponential Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $3^4 \cdot 3^2$</td>
<td>$\frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3^2}$</td>
</tr>
<tr>
<td>b. $x^3 \cdot x^5$</td>
<td>$x^8$</td>
</tr>
<tr>
<td>c. $(1.05)^3 \cdot (1.05)^4$</td>
<td>$(1.05)^7$</td>
</tr>
<tr>
<td>d. $10^4 \cdot 10^6$</td>
<td>$10^{10}$</td>
</tr>
</tbody>
</table>

2. Compare the exponents in each final expression you got in Step 1 to the exponents in the original product. Describe a way to find the exponents in the final expression without using expanded form. (i.e. a shortcut).

3. Generalize your observations in Step 2. $b^n \cdot b^n = b^{n+n} = b^{2n}$

Try these problems:

<table>
<thead>
<tr>
<th>$x^4 \cdot x^4$</th>
<th>$w^3 \cdot w^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^4 \cdot x^3$</td>
<td>$k^4 \cdot k^{-2}$</td>
</tr>
<tr>
<td>$3x^4 \cdot 5x^5$</td>
<td>$4h^2 \cdot 2h^3$</td>
</tr>
</tbody>
</table>

Division and Exponents

*Developing Exponent Laws*

1. Rewrite each product below in expanded form and then rewrite it in exponential form with a single base.

<table>
<thead>
<tr>
<th>Expanded Form</th>
<th>Exponential Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $\frac{3^4}{3^3}$</td>
<td>$\frac{3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3}$</td>
</tr>
<tr>
<td>b. $\frac{x^4}{x^2}$</td>
<td>$x^2$</td>
</tr>
<tr>
<td>c. $\frac{(1.05)^3}{(1.05)^2}$</td>
<td>$(1.05)^1$</td>
</tr>
<tr>
<td>d. $\frac{10^8}{10^7}$</td>
<td>$10^1$</td>
</tr>
</tbody>
</table>

2. Compare the exponents in each final expression you got in Step 1 to the exponents in the original product. Describe a way to find the exponents in the final expression without using expanded form. (i.e. a shortcut).

3. Generalize your observations in Step 2. $\frac{b^m}{b^n} = b^{m-n}$

4. Use the division property of exponents to rewrite each of these expressions with a single exponent.

<table>
<thead>
<tr>
<th>$\frac{y^2}{x^2}$</th>
<th>$\frac{3^3}{3^2}$</th>
<th>$\frac{7^4}{7^3}$</th>
<th>$\frac{x^3}{x^2}$</th>
<th>$\frac{x^4}{x^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{z^3}{z^2}$</td>
<td>$\frac{2^3}{2^2}$</td>
<td>$\frac{x^3}{x^2}$</td>
<td>$\frac{m^3}{m^2}$</td>
<td>$\frac{5^4}{5^3}$</td>
</tr>
</tbody>
</table>
Exponent Laws Groups & Lessons

Thursday November 5, 2014

Block 3

Kelsey  Derek
  C. XXI
  Work on MC 7.8.
  Brianna  Felix

Taeva  Samantha
  A. Exponent Laws
  Sophie  Michael

Isabella  Jonah
  C. XXI
Phoebe  Caylin

Rachel  Gina
  C. XXI
  May need in work on MC 7.8.
Sydney  Connie

Evan  Sam
  A. Exponent Laws
Sinead  Owen

Sonya  Livvy
  R. XXI
  Three More Exponent Laws
Brendan  Matt

Not here: Chris B

Block 1

Candice  Jarod
  A. Exponent Laws
  Jonathan  Sean

Phoebe  Kelly
  C. XXI.
Hannah  Riley

Shelley  Christina
  A. Exponent Laws
  Myle  Chris

Lucy  Caitlin
  MC 7.8.
Colin

Kamil  Carl
  A. Exponent Laws
Grace  Tom  Avery

Emma  Sarah
  C. XXI
Elise  Scott

Not here: Jill

Today's Task

Group A
1. Develop The Following Exponent Laws
   Power Law
   Zero Power Law
   Negative Exponent Law
2. IXL Practice on ALL exponent Laws

Group B
1. IXL Practice on F8 & F9 OR F10
2. Develop The Following Exponent Laws
   Power Law
   Zero Power Law
   Negative Exponent Law
3. IXL Practice on ALL exponent Laws

Group C
1. IXL Practice on F8 & F9 OR F10
2. Develop The Following Exponent Law
   Zero Power Law
   Power Law
3. IXL Practice on F11
Influenza Epidemic

A community of 1,000 people is assumed to be homogeneously mixed. Some individuals had returned from another community with the influenza virus (the flu). Currently there are 25 people with the flu. Assume the community has not had flu shots and all are susceptible and the flu spreads at a rate of 30% per day.

1. Clearly define the variables in your equation.
   \[ x = \quad y = \]

2. Develop an equation to model the spread of the flu.

   How do you know that the equation is exponential?

3. Make a table showing the predicted # of people who contract the flu over the first 5 days.

   How do you know that the table is exponential?

4. Use your calculator to graph your equation and determine how many days ago the initial 4 infected people returned to the community. Sketch that graph here and show the key points found on your calculator.

   How do you know that the graph is exponential?

5. How can you use your model to determine how many people will be infected 3 days from now?

6. In how many more days will 1/4 of the community be infected?
The Russian Nesting Doll is an intriguing toy for all ages. In this challenge you will determine if your nesting dolls are best modeled with an exponential function or a linear function (or some other function) and make a convincing argument for your choice.

**Expectations:**

- You will work with one set of nesting dolls.
- Collect data on the dolls.
- Create a function that models the growth/decay of your set of dolls.
- Provide EVIDENCE that your dolls are best modeled with this type of function and this function in particular.
  - Your EVIDENCE should include things like graphs, tables, equations, diagrams and you should use these representations to SHOW the reader why you believe that your claim is best.

There are no further specific guidelines. This is a pretty open ended assignment and your job is basically to convince me and do so by clearly communicating and supporting your claims.

**The Write-Up Should Include:**

- **Problem Statement**
- **Givens**
  - Including the identity of your set of Nesting Dolls and the dimension(s) you are focusing on.
- **Solution**
  - Where you determine the function type, convince me, and then create the actual equation that models the growth/decay of your dolls
- **Verification**
  - Show me that your function type makes the most sense and show that your equation works
# MiCrO # 5 Nesting Doll Challenge

**Exponential or Linear or ... ?**

*You Decide*

**Skill Assessed: Mathematical Problem Solving**
**Due: Monday, 5/12/2015**

## Mathematical Problem Solving

### Understood the Problem
- The goal of the problem is clearly identified.
- You state givens, key factors, and formulas you will use.

<table>
<thead>
<tr>
<th>Progress Levels</th>
<th>Just Beginning</th>
<th>Approaches</th>
<th>Meets</th>
</tr>
</thead>
<tbody>
<tr>
<td>JB</td>
<td>A</td>
<td>M</td>
<td></td>
</tr>
</tbody>
</table>

### Devised a solution plan
- You show a clear process for solving the problem with reasons for decisions you make.
- You have equations, graphs, and tables that lead to the solution of your problem.

<table>
<thead>
<tr>
<th>Progress Levels</th>
<th>Just Beginning</th>
<th>Approaches</th>
<th>Meets</th>
</tr>
</thead>
<tbody>
<tr>
<td>JB</td>
<td>A</td>
<td>M</td>
<td></td>
</tr>
</tbody>
</table>

### Carried out the solution plan correctly
- You collect and display data.
- You develop equations to model the scenario.
- You provide a convincing argument as to why your nesting dolls are exponential or linear or other.

<table>
<thead>
<tr>
<th>Progress Levels</th>
<th>Just Beginning</th>
<th>Approaches</th>
<th>Meets</th>
</tr>
</thead>
<tbody>
<tr>
<td>JB</td>
<td>A</td>
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<td></td>
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</tbody>
</table>
MiCrO #5 Write Up Format

Title:

Goal/Purpose:

Givens/Key Factors:
- Which Nesting Dolls are you using?
- Which dimensions are you measuring?
- Define variables (x & y)
- List the Key Features of an Exponential Function
- List the Key Features of a Linear Function
- List the Key Features of a Quadratic Function (if you have studied this and consider this function type)

Solution
- **Data:** Make a data table showing your measured data

  - **Linear or Exponential?**
    Go through the process of providing evidence for and against each type of function.

<table>
<thead>
<tr>
<th>CLAIM</th>
<th>EVIDENCE</th>
<th>REASONING</th>
</tr>
</thead>
<tbody>
<tr>
<td>A ______ Function is a better model than a ______ Function to model my Nesting Dolls.</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
• **Equation**
  - Once you have established the TYPE of function, then show step by step, how you determined your equation. You will have to show and explain how you developed each part of the equation from your data and then pull it all together into an equation.

• **Verification**
  - Use your equation or a table or graph to determine the expected dimension of the dolls and see if they come close to the actual dimensions.
Division 2 MST
MiCrO #4...Modeling Exponential Functions

Problem Solving Template Due: ________________

Video Presentation or Technology Gadget Due: ________________

The possibilities (and the outcomes) of nearly all scientific and mathematical situations rely on parameters. Understanding what the parameters are and the effects that they have help us to predict the outcomes and explain why they happen. One of our goals in this unit is to become familiar with exponential functions (growth and decay) and to be able to model any exponential situation we encounter to make predictions.

Your Task:
- Solve an exponential application problem (Mathematical Problem Solving)
- Teach your problem to others in the form of a video presentation OR technological gadget (Technical Communication) (Technology)

OPTIONS

A. Technical Communication with Mathematical Problem Solving
1. Solve your problem using the accompanying template. Turn that in for assessment.
2. Create your Presentation which will be in video format:

Presentation Requirements:
- Describe your scenario
- Present mathematical representations of your scenario using graphs, tables, and equations.
- Explain what meaning the following have in the context of your scenario and how you use them to develop an exponential equation for your scenario:
  - the parameters a and b
  - the variables x and y
- Demonstrate how you know your scenario is indeed exponential (from the graph, the table, the equation, AND the written scenario itself).
- Show how to solve your guiding questions.
- Make observations about your scenario that are interesting or different from what people might expect. Ask your own questions, and seek their answers. Try to explain WHY things are happening, not just what happens.
- Present a "demo" of your scenario or some way to show the type of growth or decay exhibited by your problem.

B. Technology with Mathematical Problem Solving
**"Only if Problem Solving is completed No Later than Monday or Tuesday."**
1. Solve your problem using the accompanying template. Turn that in for assessment.
2. Create an interactive technological learning tool (applet) that:
   a. Has A Title
   b. Has an Overview that describes the purpose of the gadget and general instructions for using the gadget.
   c. Prompts the user to adjust parameter values and explore their effect on the graph.
   d. Supplies answers so that the user can check their results with what they should see.
   e. Utilizes a variety of tools (sliders, check boxes, input boxes, animation)
   f. Has general equation on screen that changes as the user changes the parameters.
   g. Utilize deliberate and appropriate ranges for parameter values... (i.e. b is not negative, there are sufficient fractional "b" values allowed on sliders and input boxes. You do NOT leave these values to defaults.)

Using sliders, movement buttons, and animations, you will model your scenario and show how the parameters of an exponential function affect the exponential graph.
Samples of what an interactive tool can look like will be up on our class pages.

3. Articulate the decisions you made in a reflection.

- Why was the technology you chose the right technology for the task?
- What did you already know about this technology, and what specific features did you need to learn in order to execute your vision?
- Why/how did you decide on the particular design you created?
- How does the tech-based demonstration of the exponential function communicate more effectively than a traditional paper? Where is it less effective?
- What is happening "behind the scenes" that would not be readily apparent to someone viewing your work? For example:
  - How did you adjust sliders to give the range of values you selected?
  - Describe any script you had to write for check boxes, animation, etc.
**MiCrO #4**

**Modeling Exponential Functions**

*Video Organizational Template*

Summarize your scenario in your own words (*Be sure to highlight important Givens)*:

Create a data table & graph for your scenario.

<p>| | | | |</p>
<table>
<thead>
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</tr>
</tbody>
</table>

*Insert Graph Here*

- What does it represent in your scenario?
- What does x represent in your scenario?
- What does y represent in your scenario?

How can you tell your scenario is exponential...
- From the graph? (*Include the graph as you describe and show the key features.)*
- From the table? (*Include the table as you describe and show the key features.)*
- From the equation(s)? (*Include the equation as you describe and show the key features.)*
- From the written description? (*Be specific as to what words tell you it is exponential.)*

Answer your guiding questions. Show how to answer them using at least 2 methods each (graph, table, equation).

Write an explicit rule (*y = a*b*) to describe the relationship between your variables:

Describe and Show how that equation is developed...*remember the debriefing that we did with Alice!*

- What is the value of the coefficient, \(a\)?
- What does it represent in your scenario?

- What is the value of the base, \(b\)?
- How will you model your scenario? (for Technical Communication only)

Make some observations about your scenario that are special, different, or interesting. Try to explain why they occur.
1. The Birthday Gift
Grace’s mother decided to give her daughter a very special 16th birthday gift. Her mom would give her $1.00 on her 16th birthday, January 3rd. Then, Grace would receive 4 times that amount on February 1st, 1 month later. For every month after that, she would receive 4 times what she had received the month before.

- Create a demonstration that models your scenario, including a summary of the situation. If you can’t physically demonstrate the scenario, you may need to be creative in your demonstration.
- Model the situation using graphs, tables, and equations (recursive and explicit).
- Explain how you know that your scenario is exponential, based on your graph, equation, table, and description of the scenario.
- Describe the relevance of the coefficient (a), the base (b), the exponent (x) and the y value in your scenario.
- Make observations about your situation that are interesting or unique to your situation. Explain why they happen, when possible.

a. How much money would Grace receive on Oct. 3rd?
b. After how many months, would Grace be receiving over $500.00 a month?

4. Keep Growing!!
Doctors use growth charts to determine whether a child’s height is in the normal range. Percentiles show how far from a measurement is from the normal range. For children who fall under the 3rd percentile level on these charts, a growth hormone may be used to help them grow at a more normal rate. If 10 milligrams of one particular growth hormone is introduced to the bloodstream, as much as 70% will still be present the next day. After another day, 70% of that amount will remain, and so on.

- Create a demonstration that models your scenario, including a summary of the situation. If you can’t physically demonstrate the scenario, you may need to be creative in your demonstration.
- Model the situation using graphs, tables, and equations (recursive and explicit).
- Explain how you know that your scenario is exponential, based on your graph, equation, table, and description of the scenario.
- Describe the relevance of the coefficient (a), the base (b), the exponent (x) and the y value in your scenario.
- Make observations about your situation that are interesting or unique to your situation. Explain why they happen, when possible.

Use your tables, graphs and equations to show how to find the answers to the following questions:

a. How much growth hormone will remain in the bloodstream after 5 days?
b. How long will it take for the original 10-milligram does to be reduced to 0.1 milligram?

6. Nigerian Population
Nigeria is the most populous country in Africa. Its 1995 population was about 140 million (110 * 10^6). It is growing at a rate of about 3.2% per year.

- Create a demonstration that models your scenario, including a summary of the situation. If you can’t physically demonstrate the scenario, you may need to be creative in your demonstration.
- Model the situation using graphs, tables, and equations (recursive and explicit).
- Explain how you know that your scenario is exponential, based on your graph, equation, table, and description of the scenario.
- Describe the relevance of the coefficient (a), the base (b), the exponent (x) and the y value in your scenario.
- Make observations about your situation that are interesting or unique to your situation. Explain why they happen, when possible.

Use your tables, graphs and equations to show how to find the answers to the following questions:

a. If the growth rate continues at 3.2%, what will the population of Nigeria be in the year 2005?
b. When will Nigeria reach a population of 200 million?

8. Buying A Car?
Joe purchased a new car for $21,500. The value of the car decreases by 20% each year (called depreciation)

- Create a demonstration that models your scenario, including a summary of the situation. If you can’t physically demonstrate the scenario, you may need to be creative in your demonstration.
- Model the situation using graphs, tables, and equations (recursive and explicit).
- Explain how you know that your scenario is exponential, based on your graph, equation, table, and description of the scenario.
- Describe the relevance of the coefficient (a), the base (b), the exponent (x) and the y value in your scenario.
- Make observations about your situation that are interesting or unique to your situation. Explain why they happen, when possible.

Use your tables, graphs and equations to show how to find the answers to the following questions:

a. What is the value of the car after nine years?
b. How long until the car is worth only $10,000?
15. The Sierpinski Carpet Fractal

One of the most interesting and famous fractal patterns is named after the Polish mathematician Wacław Sierpiński. The fractal is made by following these steps for making the fractal:
1) Start with 1 square meter of carpet.
2) Divide the shaded square into 9 congruent smaller squares.
3) "Cut-out" the middle square.
4) Repeat steps 2 and 3 for each shaded square.

The first 3 stages of this fractal are shown below.

### E3. Carbon 14 Dating

Carbon 14 is a isotope of carbon that is formed when radiation from the Sun strikes ordinary carbon dioxide in the atmosphere. Plants such as trees, which get their carbon dioxide from the atmosphere, therefore contain small amounts of carbon 14. Once a particular part of a plant has been formed, no more new carbon 14 is taken in. The carbon 14 in that part of the plant decays slowly, transmuting into nitrogen 14. Let P be the percent of carbon remaining in a part of a tree that grew 1 year ago.

- Write an equation that models the percentage of Carbon-14 remaining over time. You may assume that the "half-life" of carbon 14 is 5700 years, meaning that 50% of carbon 14 present when t = 0, only 25% remains when t = 5750 years.

- Create a demonstration that models your scenario, including a summary of the situation. If you can't physically demonstrate the scenario, you must need to be creative in your demonstration.
- Model the situation using graphs, tables, and equations.
- Explain how you know that your scenario is exponential, based on your graph, equation, table, and description of the scenario.
- Describe the relevance of the coefficient (k), the base (b), the exponent (n) and the y value in your scenario.
- Make observations about your situation that are interesting or unique to your situation. Explain why they happen, when possible.
- Use your tables, graphs and equations to show how to find the answers to the following questions:

  a. How much carpet remains at the 10th stage?

  b. At what stage will these be less than 1 cm² of carpet remaining?

(1 cm² = 0.0001 m²)
These are Extensions that explore the more general form of an exponential function in more depth.

**Exponential**

1. \( y = 2^x \)
2. \( y = 3^x \)
3. \( y = 4^x \)

**Asymptote**

Due: Thursday, April 11

Recently, you investigated trends in exponential functions. Your challenge now is to clearly communicate one aspect of what you learned in a concise, elegant, visually rich written report that presents a mathematical argument for your claim. (An example of what this might look like is attached.)

Your topic:

- When \( d > 1 \), what is the effect on the exponential graph? \( y = a \cdot \frac{1}{d^{d-x}} + c \)?
  - Show 3 sample graphs (keeping all parameters constant except for "d"), and describe the effects seen on the graphs, labeling clearly.
  - Algebraically explain why these effects make sense when "d" > 1.
  - Consider how you can use exponential laws to explain the effects of "d".

Your write up should take on the following format

- Make a bold claim
- Illustrate the claim with examples (graphs, tables, etc)
- Describe the patterns or features
- Justify the claim with a mathematical argument

---

**Exponential**

1. \( y = 2^x \)
2. \( y = 3^x \)
3. \( y = 4^x \)

**Asymptote**

Due: Thursday, April 11

Recently, you investigated trends in exponential functions. Your challenge now is to clearly communicate one aspect of what you learned in a concise, elegant, visually rich written report that presents a mathematical argument for your claim. (An example of what this might look like is attached.)

Your topic:

- When \( a > 0 \), what is the effect on the exponential graph? \( y = a \cdot \frac{1}{d^{d-x}} + c \)?
- When \( a < 0 \), what is the effect on the exponential graph? \( y = a \cdot \frac{1}{d^{d-x}} + c \)?
  - Show 3-5 sample graphs (keeping all parameters constant except for "a"), and describe the effects seen on the graphs, labeling clearly.
  - Algebraically explain why these effects make sense when "a" changes.
  - Consider how you can use exponential laws to explain the effects of "a".

Your write up should take on the following format

- Make a bold claim
- Illustrate the claim with examples (graphs, tables, etc)
- Describe the patterns or features
- Justify the claim with a mathematical argument
Describe and show how to develop an equation to model this scenario.

What is the value of the coefficient, \( a \)?

What does it represent in your scenario?

Where do you find it on your data table?

What is the value of the base, \( b \)?

What does it represent in your scenario?

How do you get the \( b \)-value from the given information?

How do you get it from your data table?

What does \( x \) represent in your scenario?

What does \( y \) represent in your scenario?

How can you tell your scenario is exponential...

... From the graph? (Include the graph as you describe and show the key features.)

b. In what year was the federal debt \$2,120?

<table>
<thead>
<tr>
<th>Using Table</th>
<th>Using Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

State your answer to question:

Guiding Questions: (Show how to answer these using at least 2 methods each (graph, table, equation))

a. What was the federal debt in 1980?

<table>
<thead>
<tr>
<th>Using Equation</th>
<th>Using Table or Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

State your answer to question "a":
Write the Equation that Models this scenario:
(Describe how you determined each of the parameters to derive the equation that you did.)

Verify that your equation works: (by demonstrating that 2 of the points on the table AND graph match those given by the equation.)

<table>
<thead>
<tr>
<th>Test Point #1</th>
<th>Test Point #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(not x = 0 or x = 1)</td>
<td>(not x = 0 or x = 1)</td>
</tr>
</tbody>
</table>

How can you tell your scenario is exponential... ...From the graph? (Include the graph as you describe and show the key features.)

...From the table? (Include the table as you describe and show the key features.)

... From the equation(s)? (Include the equation as you describe and show the key features.)

Guiding Question #1: (Restate your first question and then show your solution using at least 2 representations: equation, graph, table) Paste your tables, and graphs in this spot and be sure to show how the table or graph give you the solution.)
Guiding Question #2: Precise your 2nd question and then show your solution using at least 2 representations (equation, graph, table). Paste your tables, and graphs in this spot and be sure to SHOW how the table or graph give you the solution.